UNIVERSITY OF ZAGREB
FACULTY OF CIVIL ENGINEERING
STRUCTURES DEPARTMENT

A Probabilistic Analysis of the Solid Timber Structure (Benchmark example)

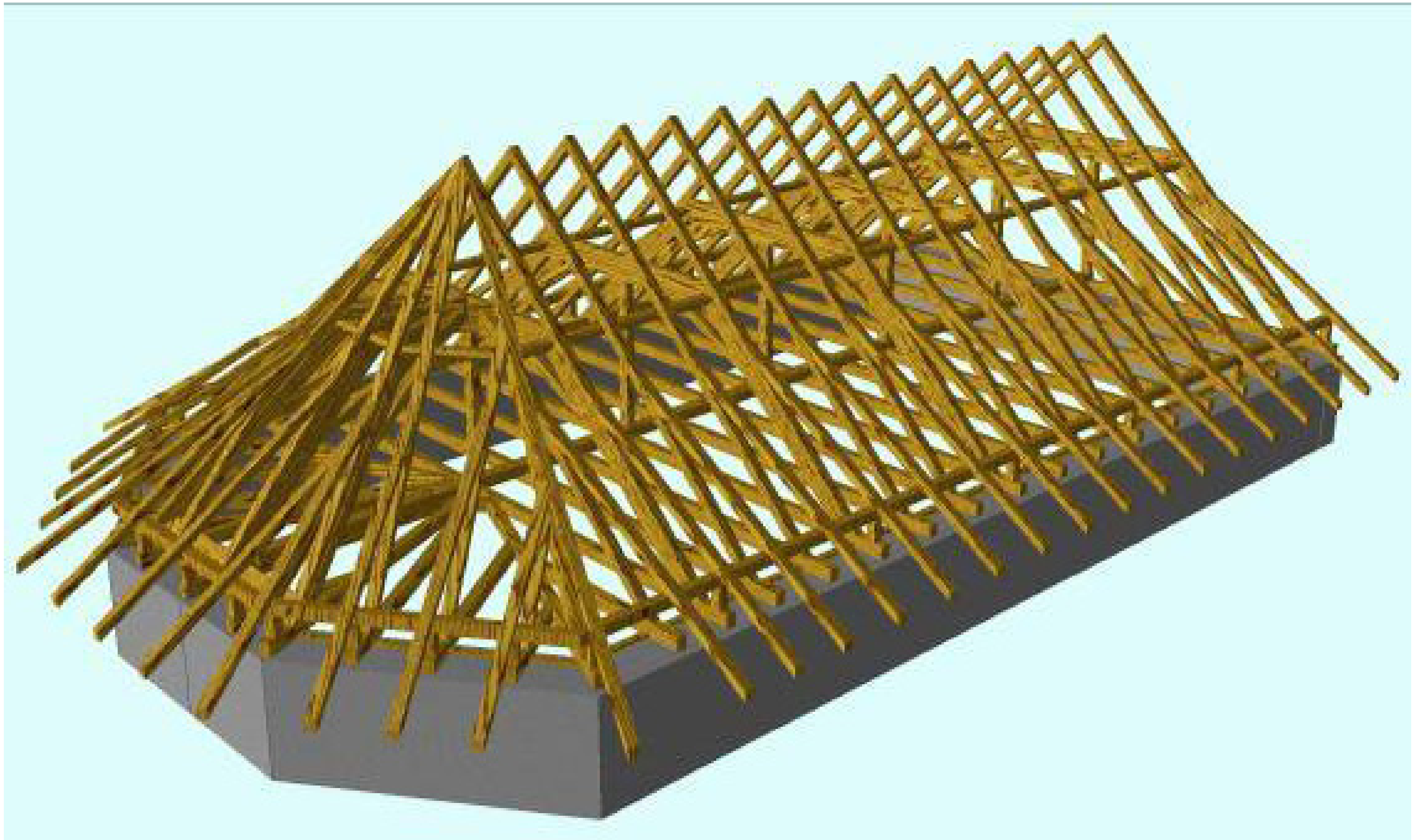
1st Draft

Dean Čizmar,
Structural Department, Faculty of Civil Engineering

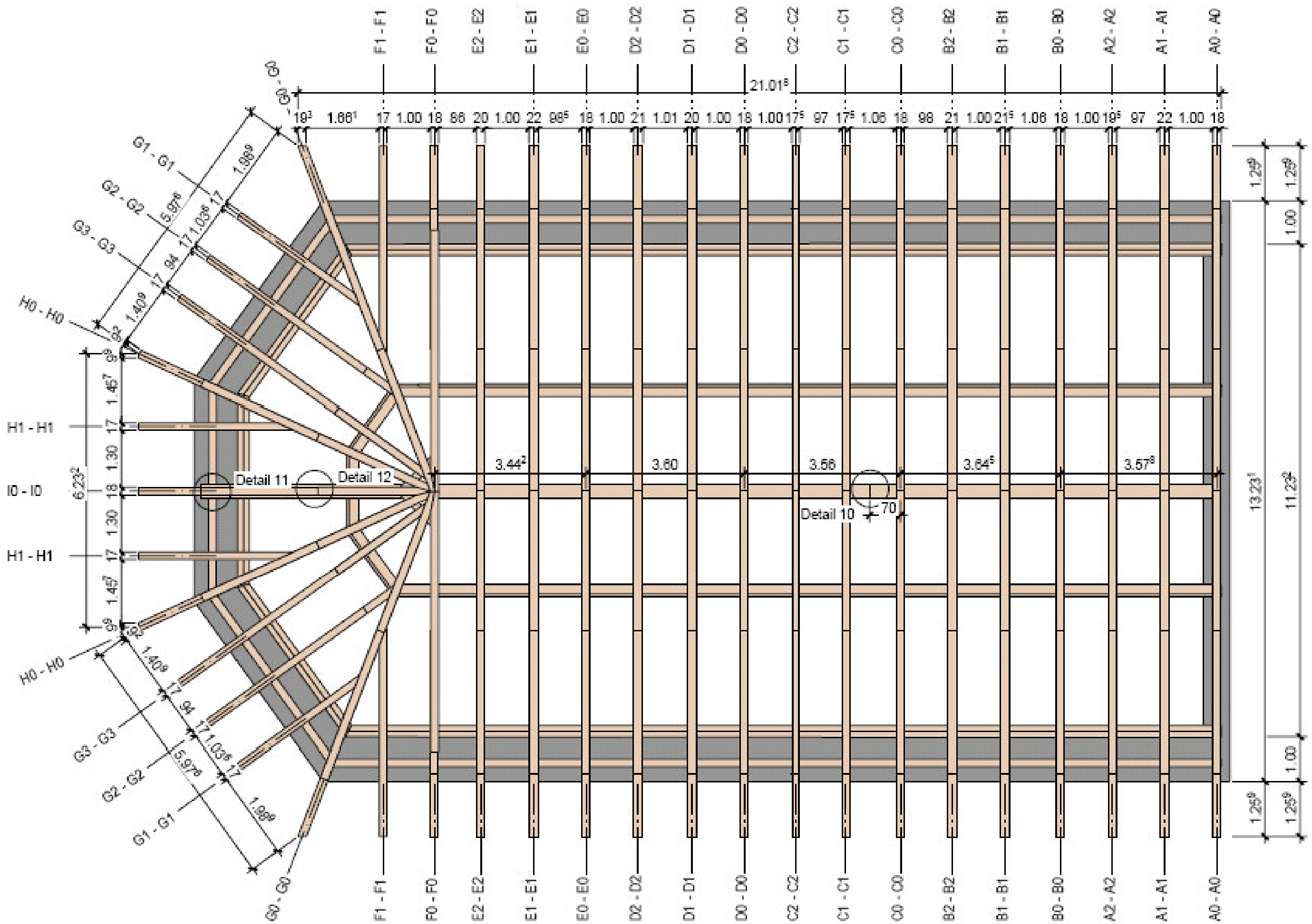
TOC

STRUCTURE

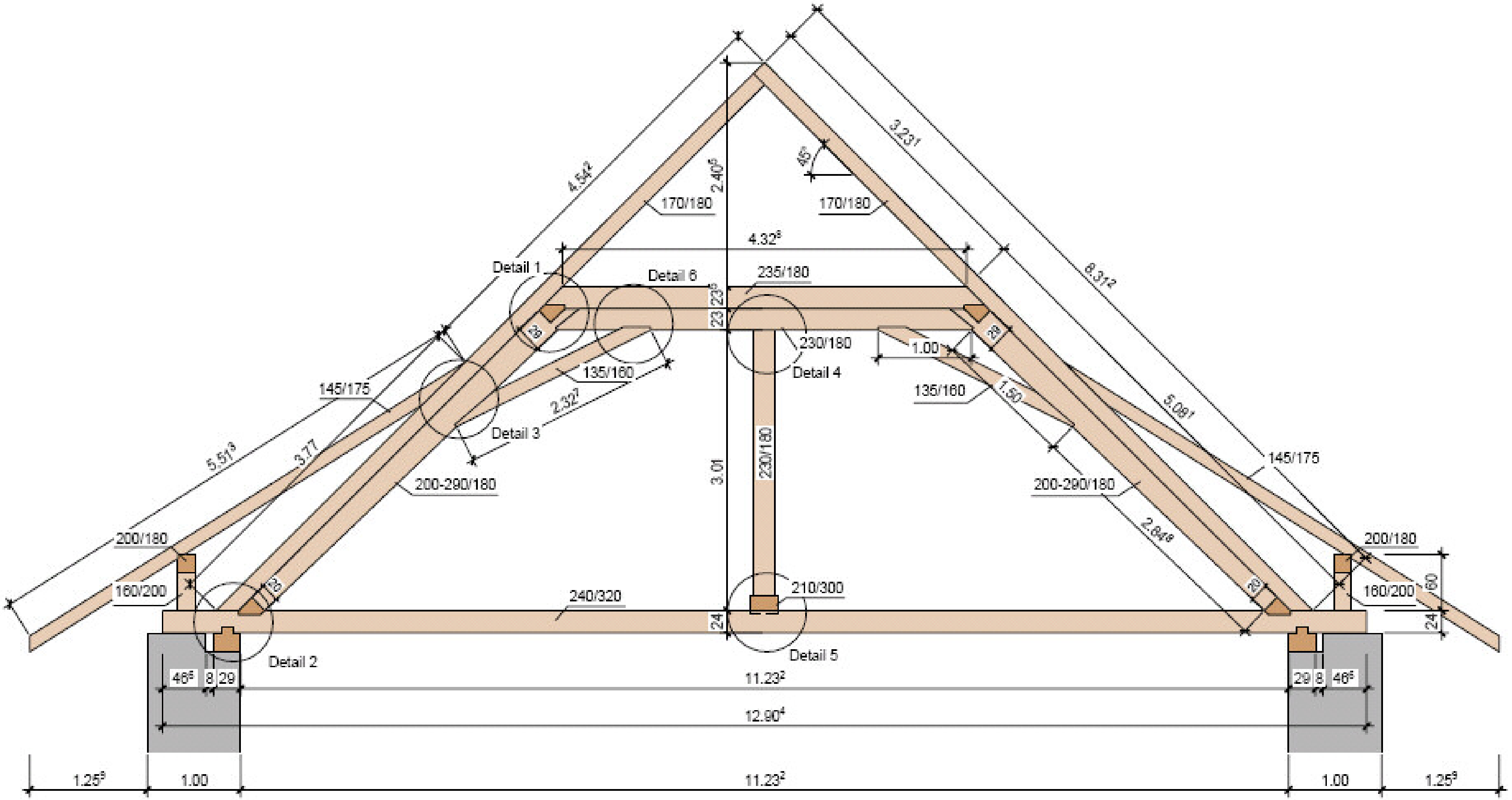
- Church in the village of Krauchtal in Switzerland
- 300 years old church
- no building codes



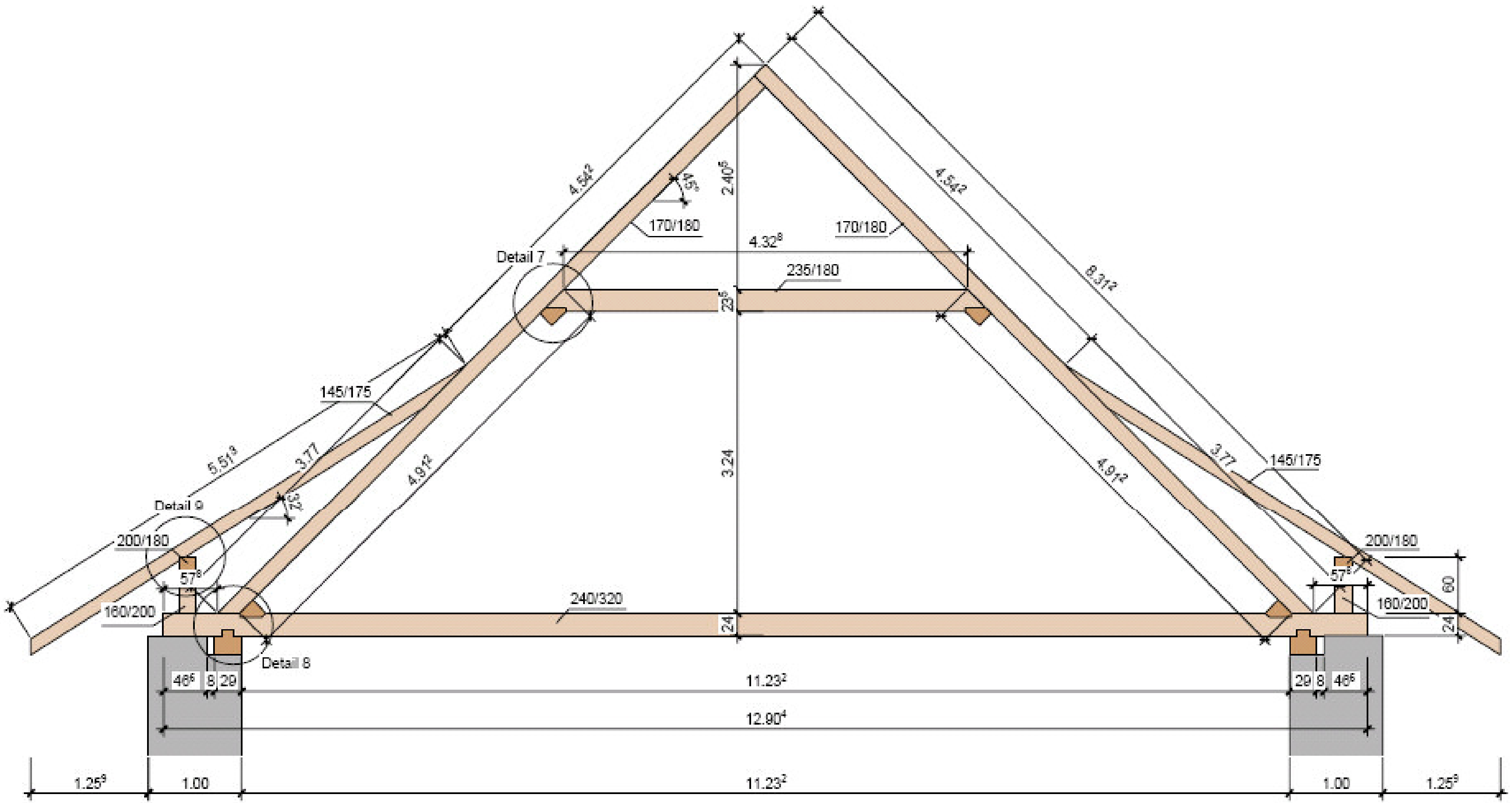
3d model of the church



Layout of the structure

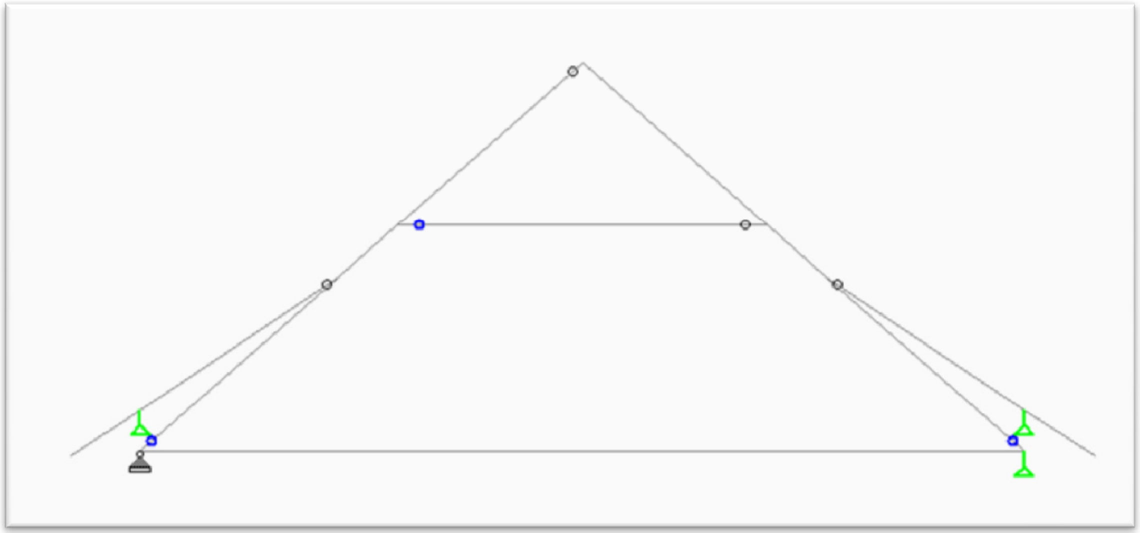

Detail 1
Detail 2
Detail 3
Detail 4
Detail 5
Detail 6
170/180
170/180
235/180
230/180
135/160
135/160
145/175
145/175
200-290/180
200-290/180
200/180
200/180
160/200
160/200
240/320
210/300
230/180
4.54^{2}
3.23^{1}
8.31^{2}
5.08^{1}
2.84^{8}
5.51^{3}
3.77
2.32^{7}
4.32^{8}
2.40^{5}
3.01
1.00
1.50
45°
46^{5}
8
29
11.23^{2}
12.90^{4}
1.25^{9}
1.00
24
60

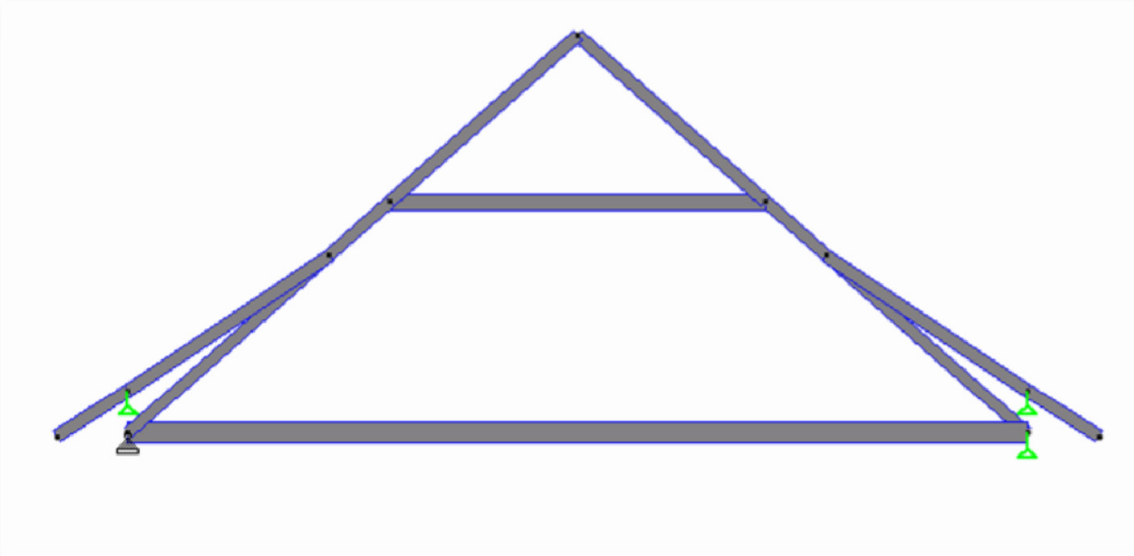
Main frame



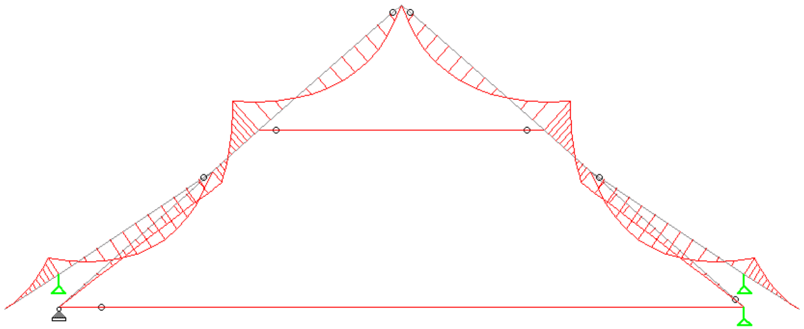
Minor frame
(B1-B1)

MODEL OF STRUCTURE

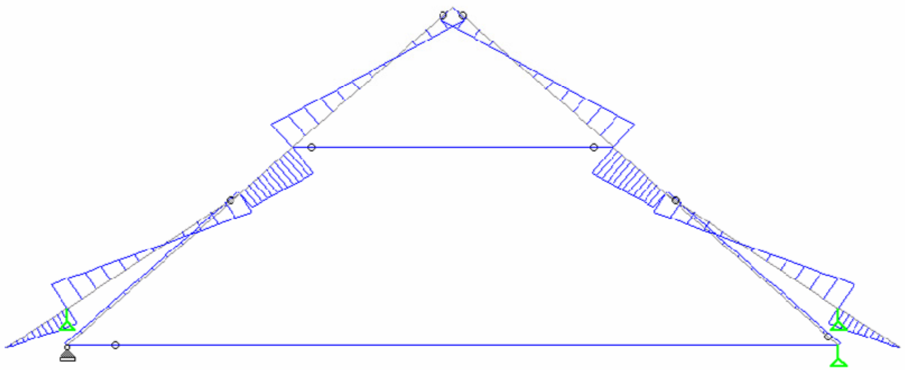


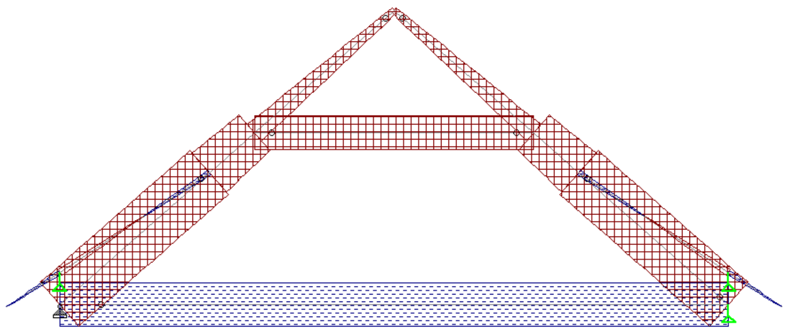


Mechanical model of the minor frame & cross sections



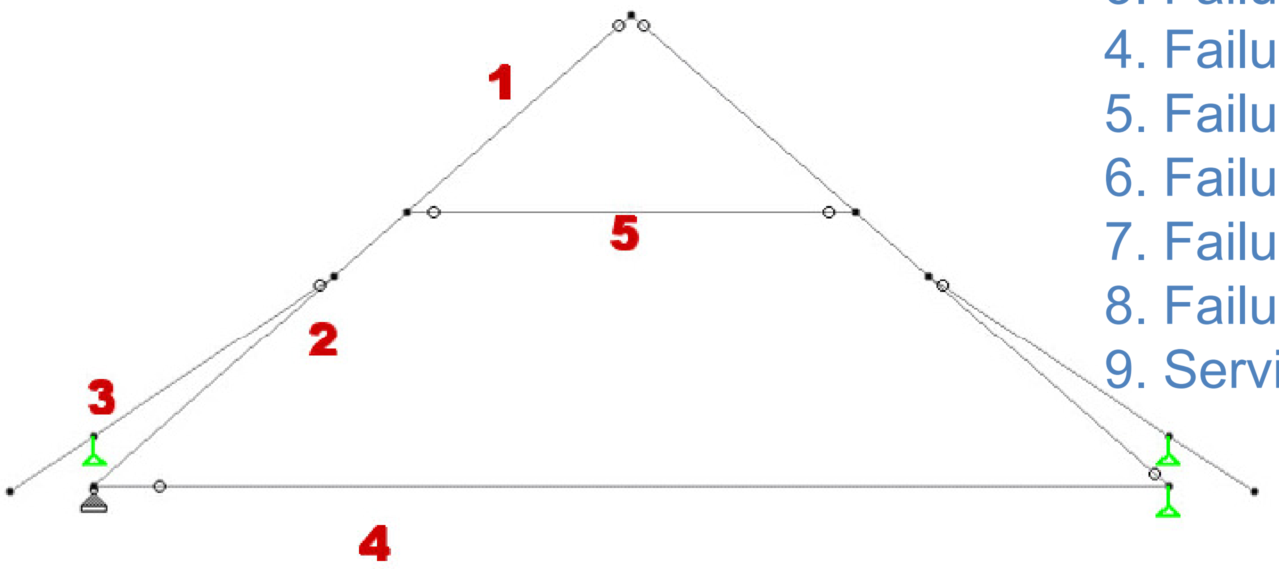
Bending moments, shear and axial forces (snow load)





UNIVERSITY OF ZAGREB
FACULTY OF CIVIL ENGINEERING
STRUCTURES DEPARTMENT

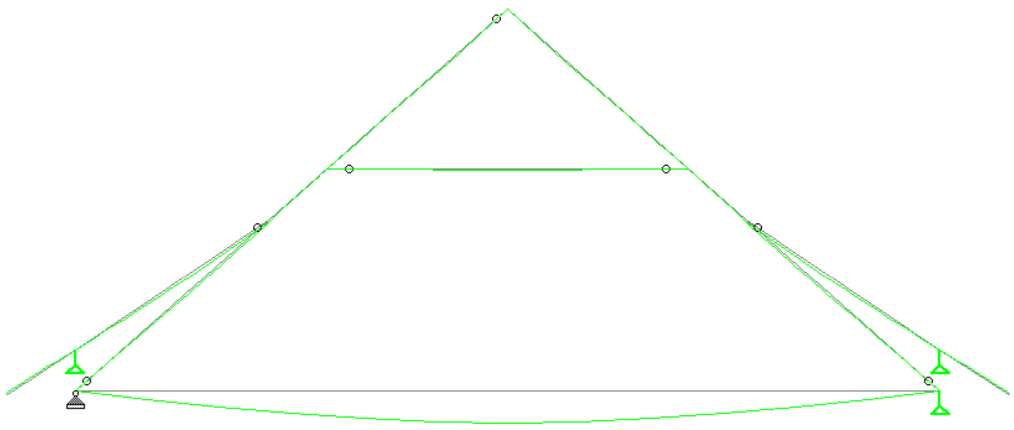
PROBABILISTIC MODEL



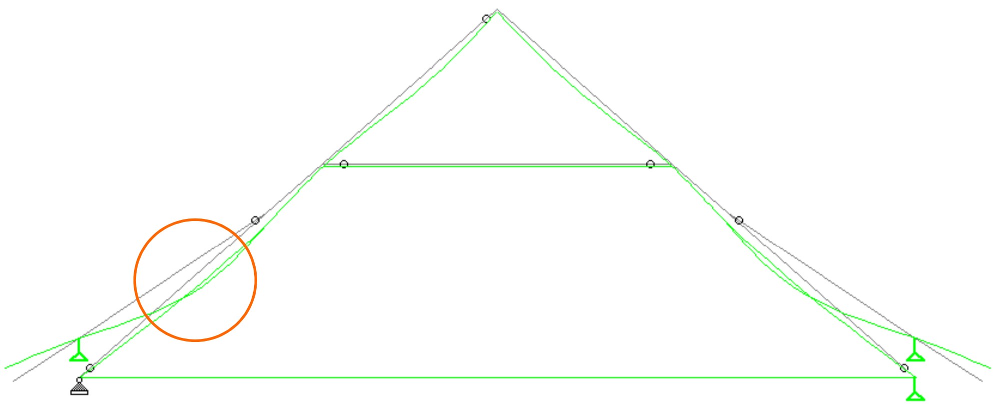
Structural elements

FAILURE MODES

1. Failure in element 1 (V)
2. Failure in element 3 (V)
3. Failure in element 1 (N+M)
4. Failure in element 2 (N+M)
5. Failure in element 3 (N+M)
6. Failure in element 4 (N+M)
7. Failure in element 5 (N)
8. Failure in element 4 (CP)
9. Serviceability limit state



Vertical displacements due to permanent load



Vertical displacements due to snow load

PROBABILISTIC VARIABLES ASSUMED FOR FORM ANALYSIS

Variable	Distribution	Expected value	COV	Designation
f_m [Mpa]	LN	31,4	0,25	Bending strength
$f_{c,o}$ [Mpa]	LN	$5f_m^{0,45}$=23,6	0,20	Compression strength
f_v	LN	$0,2\ f_m^{0,8}$	0,25	Shear strength
f_t	LN	0,015E [ρ_{den}]	2,5COV[ρ_{den}] =0,25	Tensile strength
$f_{c,90}$	N	0,008 E[ρ_{den}]	COV [ρ_{den}]=0,1	Compressive strength ⊥ to grain
X	LN	1	0,05	Model uncertainty
ρ_{den}	N	500	0,1	Timber density
S [kn/m^2]	G	0,43	0,5	Snow load (value on ground)
G [kn/m^2]	N	0,80	0,10	Permanent load

Probabilistic variables

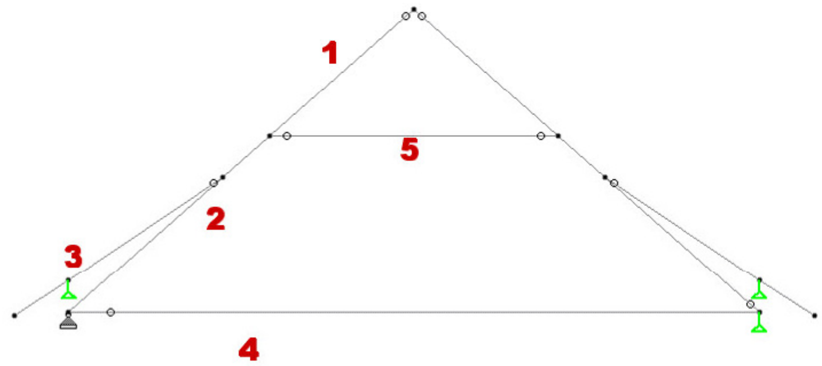
Variable	Distribution	Expected value	COV	Designation
L	D	1,1	-	Distance between bearing structure
k_{mod}	D	0,8	-	Modification factor
b	N	b_{nom}	0,07	Width of the beam
h	N	h_{norm}	0,07	Height of the beam
b_{len}	N	b_{len}	0,10	Contact length
k_c	D	k_c	-	Instability coefficient (N)
k_{crit}	D	k_{crit}	-	Instability coefficient (M)
δ_L	D	17 mm	-	Deflection limit

Probabilistic variables (cont.)

RESULTS (RELIABILITY OF COMPONENTS)

- FORM analysis is made for each of the limit state functions

  1. Failure in element 1 (V)
  2. Failure in element 3 (V)
  3. Failure in element 1 (N+M)
  4. Failure in element 2 (N+M)
  5. Failure in element 3 (N+M)
  6. Failure in element 4 (N+M)
  7. Failure in element 5 (N)
  8. Failure in element 4 (CP)
  9. Serviceability limit state



Cost of safety	Minor consequences of failure	Moderate consequences of failure	Large consequences failure
Large (A)	$\beta = 3.1$ ($P_f \approx 10^{-3}$)	$\beta = 3.3$ ($P_f \approx 5 \cdot 10^{-4}$)	$\beta = 3.7$ ($P_f \approx 10^{-4}$)
Normal (B)	$\beta = 3.7$ ($P_f \approx 10^{-4}$)	$\beta = 4.2$ ($P_f \approx 10^{-5}$)	$\beta = 4.4$ ($P_f \approx 5 \cdot 10^{-6}$)
Small (C)	$\beta = 4.2$ ($P_f \approx 10^{-5}$)	$\beta = 4.4$ ($P_f \approx 5 \cdot 10^{-6}$)	$\beta = 4.7$ ($P_f \approx 10^{-6}$)

Tentative target reliability indices β (PMC)

Failure mode	1	2	3	4	5	6	7	8	9
β	12,6	11,8	10,2	8,48	7,73	7,01	13,8	9,56	4,97

Reliability indices

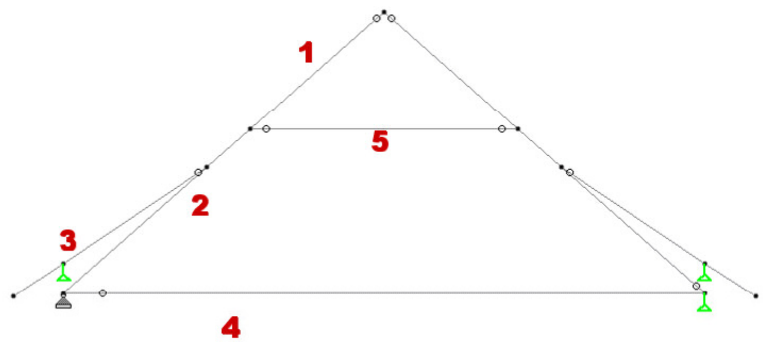
- very high reliability indices – WHY??

1	2	3	4	5	6	7	8	9	10	11
5.58	3.40	6.55	5.76	6.58	5.37	6.05	4.96	4.81	6.31	3.18

Indices for Norwegian sports centre
(Poul Henning Kirkegaard & John Dalsgaard Sørensen)

- big (massive) timber elements – old structure, no building codes
- closely spaced girders
- relatively small snow load
- wind actions are not considered

RESULTS (RELIABILITY OF SYSTEM)



- Series system is assumed

- Simple bounds:

$$\max_{i=1}^{m} P(M_i \leq 0) \leq P_f^s \leq \sum_{i=1}^{m} (P(M_i \leq 0))$$

$$-\Phi^{-1}\left(\sum_{i=1}^{m} (-\beta_i)\right) \leq \beta_s \leq \min_{i=1}^{m} \beta_i$$

SYSTEM RELIABILITY $\beta_s \in [6.84, 7.01]$

ROBUSTNESS

1) Ecosystems
The ability of a system to maintain function even with changes in internal structure or external environment

2) Statistics
A robust statistical technique is insensitive against small deviations in the assumptions

3) Structural Standards
The consequences of structural failure are not disproportional to the effect causing the failure
(this definitions exists in many different codes; also in Croatian building legislation)

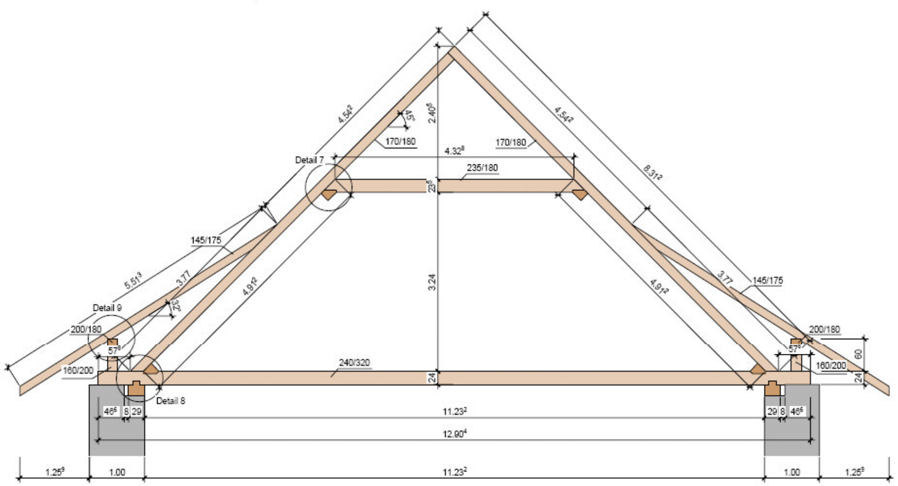
Different scenarios used to asses robustness of the structure:

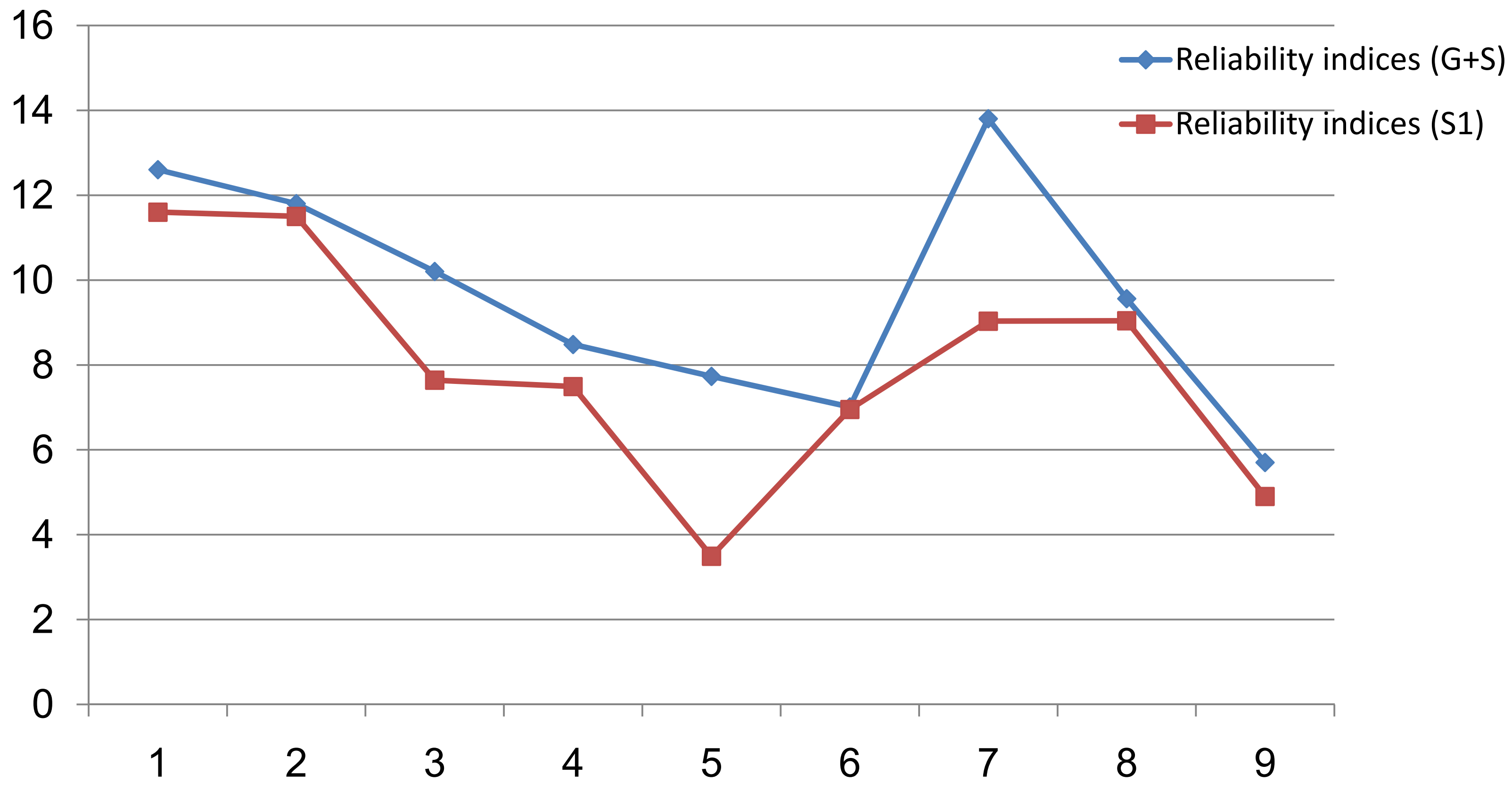
1) expected value of snow load increased by 50%, COV V=0,7 (S1)

2) 3 cm sinking of vertical support is assumed (S2)

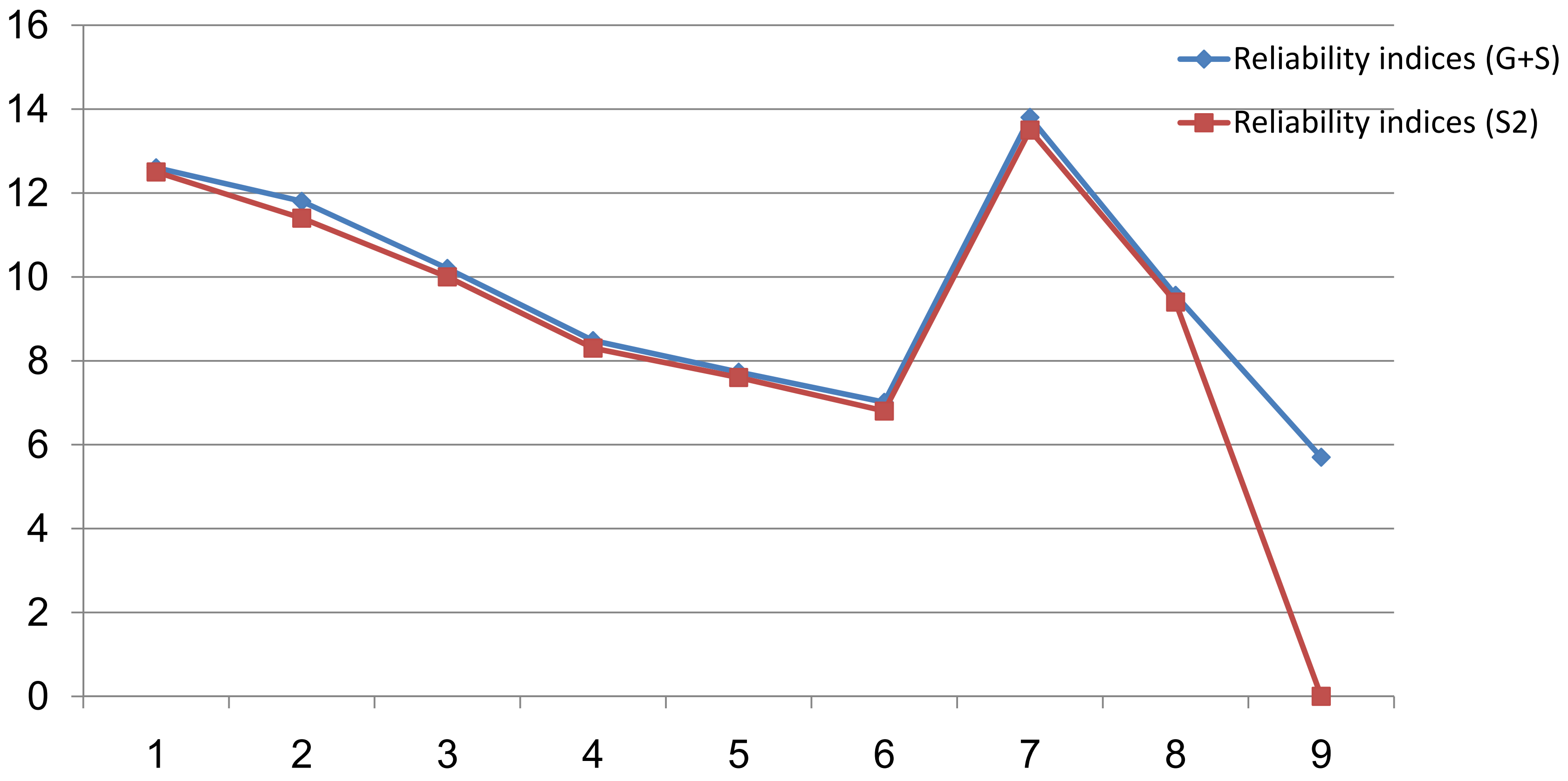
3) failure of truss element (S3)

4) failure of element 3 (S4)





Reliability indices (different failure modes and scenarios)



Reliability indices (different failure modes and scenarios)

Scenarios 3 & 4 are to be calculated

Other scenarios or key elements???

Wind loads

Modeling of joints

CONCLUSION

Simplified modeling (linear behavior in joints)

Reliability analysis indicates extremely low probability of failure (for both the ULS and SLS)

Under the increased load structure has minimal β=3,49 (ULS)

Based on two scenarious → rebust structure